
Algorithm 3 Efficient implementation of SVP, $\mathcal{K} = \{\mathbb{R}, \mathbb{C}\}$

Require: $\|\mathcal{A}\|$, $u_0 \in \mathcal{K}^{m \times r}$, $v_0 \in \mathcal{K}^{n \times r}$, $d_0 \in \mathcal{K}^r$

Require: Function $A : (u, d, v) \mapsto \mathcal{A}(u \operatorname{diag}(d) v^H)$

Require: Function $A\mathfrak{t} : (\mathbf{z}, w) \mapsto \mathcal{A}^*(\mathbf{z})w$

Require: Function $A\mathfrak{t}^H : (\mathbf{z}, w) \mapsto (\mathcal{A}^*(\mathbf{z}))^H w$

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1:  $\mu \leftarrow 4/\|\mathcal{A}\|^2$ 
2:  $v_{-1} \leftarrow 0$ ,  $u_{-1} \leftarrow 0$ ,  $d_{-1} \leftarrow 0$ 
3: for  $i = 0, 1, \dots$  do
4:   Compute  $\beta_i$  // See text
5:    $u_y \leftarrow [u_i, u_{i-1}]$ ,  $v_y \leftarrow [v_i, v_{i-1}]$ 
6:    $d_y \leftarrow [(1 + \beta)d_i, -\beta d_{i-1}]$  //  $\mathbf{Y}_{i+1} = (1 + \beta_i)\mathbf{X}_i - \beta_i\mathbf{X}_{i-1}$ 
7:    $\mathbf{z} \leftarrow A(u_y, d_y, v_y)$  // Compute the residual
8:   Define the functions
       $h : w \mapsto u_y \operatorname{diag}(d_y) v_y^H w - \mu A\mathfrak{t}(\mathbf{z}, w)$ 
       $h^H : w \mapsto v_y \operatorname{diag}(d_y) u_y^H w - \mu A\mathfrak{t}^H(\mathbf{z}, w)$ 
9:    $(u_{i+1}, d_{i+1}, v_{i+1}) \leftarrow \text{RandomizedSVD}(h, h^H, r)$ 
10: end for
11: return  $X \leftarrow u_i d_i v_i^H$  // If desired
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