Circuits and Systems I

LECTURE #10
FIR Filtering: An Introduction

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Outline - Today

• Today <> Section 5-1
  Section 5-2
  Section 5-3

• Next week <> Section 5-4
  Section 5-5
  Section 5-6
  Section 5-7

CSI Progress Level:
Lecture Objectives

• INTRODUCE FILTERING IDEA
  – Weighted Average
  – Running Average

• FINITE IMPULSE RESPONSE FILTERS
  – FIR Filters
  – Show how to compute the output $y[n]$ from the input signal, $x[n]$
Digital Filtering

- **CONCENTRATE** on the COMPUTER
  - PROCESSING ALGORITHMS
  - SOFTWARE (MATLAB)
  - HARDWARE: DSP chips, VLSI

- **DSP**: DIGITAL SIGNAL PROCESSING
The TMS32010, 1983

First PC plug-in board from Atlanta Signal Processors Inc.
For the price of a small house, you could have one of these.
Digital Cell Phone (circa. 2000)

Now it plays video
OPERATE on $x[n]$ to get $y[n]$

WANT a **GENERAL** CLASS of SYSTEMS

- **ANALYZE** the SYSTEM
  - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
- **SYNTHESIZE** the SYSTEM
D-T System Examples

- **EXAMPLES:**
  - **POINTWISE OPERATORS**
    - **SQUARING:** \( y[n] = (x[n])^2 \)
  - **RUNNING AVERAGE**
    - **RULE:** “the output at time \( n \) is the average of three consecutive input values”
Discrete-Time Signal

- $x[n]$ is a LIST of NUMBERS
  - INDEXED by "$n$"
3-PT Average System

- **ADD 3 CONSECUTIVE NUMBERS**
  - Do this for each “\( n \)”

the following input–output equation

\[
y[n] = \frac{1}{3} (x[n] + x[n + 1] + x[n + 2])
\]

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n &lt; -2 )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>( n &gt; 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x[n] )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( y[n] )</td>
<td>0</td>
<td>( \frac{2}{3} )</td>
<td>2</td>
<td>4</td>
<td>( \frac{14}{3} )</td>
<td>4</td>
<td>2</td>
<td>( \frac{2}{3} )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\( n=0 \) \( y[0] = \frac{1}{3} (x[0] + x[1] + x[2]) \)

\( n=1 \) \( y[1] = \frac{1}{3} (x[1] + x[2] + x[3]) \)
Figure 5.2  Finite-length input signal, $x[n]$.  

\[ y[n] = \frac{1}{3} (x[n] + x[n + 1] + x[n + 2]) \]

Figure 5.3  Output of running average, $y[n]$. 
Sec. 5.2 The Running Average Filter

FIR Filtering
(a weighted sum over past, present, and future points)

Figure 5.4 The running-average filter calculation at time index $n$ uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).
Another 3-pt Averager

- Uses “PAST” VALUES of $x[n]$
  - IMPORTANT IF “n” represents REAL TIME
    - WHEN $x[n]$ & $y[n]$ ARE STREAMS

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$n &lt; -2$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>$n &gt; 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x[n]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$y[n]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\frac{2}{3})</td>
<td>2</td>
<td>4</td>
<td>(\frac{14}{3})</td>
<td>4</td>
<td>2</td>
<td>(\frac{2}{3})</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
General FIR Filter

• FILTER COEFFICIENTS \{b_k\}
  - DEFINE THE FILTER
    
    \[ y[n] = \sum_{k=0}^{M} b_k x[n - k] \]

  - For example,
    
    \[ b_k = \{3, -1, 2, 1\} \]

    \[ y[n] = \sum_{k=0}^{3} b_k x[n - k] = 3x[n] - x[n - 1] + 2x[n - 2] + x[n - 3] \]
General FIR Filter

- **FILTER COEFFICIENTS** \( \{b_k\} \)

\[
y[n] = \sum_{k=0}^{M} b_k x[n - k]
\]

- **FILTER ORDER** is M
- **FILTER LENGTH** is \( L = M + 1 \)
  - NUMBER of FILTER COEFFS is L
General FIR Filter

- SLIDE a WINDOW across $x[n]$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$
Filtered Stock Signal

INPUT

OUTPUT

50-pt Averager
Special Input Signals

- $x[n] = \text{SINUSOID}$
- $x[n]$ has only one NON-ZERO VALUE

UNIT-IMPULSE

$$\delta[n] = \begin{cases} 
1 & n = 0 \\
0 & n \neq 0
\end{cases}$$

FREQUENCY RESPONSE (LATER)
UNIT IMPULSE SIGNAL $\delta[n]$

<table>
<thead>
<tr>
<th>$n$</th>
<th>...</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta[n]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\delta[n−3]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$\delta[n]$ is NON-ZERO When its argument is equal to ZERO

**Figure 5.7** Shifted impulse sequence, $\delta[n−3]$. 
MATH FORMULA for $x[n]$

- Use **SHIFTED IMPULSES** to write $x[n]$

$$x[n] = 2\delta[n] + 4\delta[n - 1] + 6\delta[n - 2] + 4\delta[n - 3] + 2\delta[n - 4]$$
### Sum of Shifted Impulses

This formula **ALWAYS** works

\[
x[n] = \sum_{k} x[k] \delta[n - k]
\]

\[
= \ldots + x[-1] \delta[n + 1] + x[0] \delta[n] + x[1] \delta[n - 1] + \ldots
\]  

(5.3.6)
4-pt Averager

- CAUSAL SYSTEM: USE PAST VALUES

\[ y[n] = \frac{1}{4} (x[n] + x[n-1] + x[n-2] + x[n-3]) \]

- INPUT = UNIT IMPULSE SIGNAL = \( \delta[n] \)

\[ x[n] = \delta[n] \]

\[ y[n] = \frac{1}{4} \delta[n] + \frac{1}{4} \delta[n-1] + \frac{1}{4} \delta[n-2] + \frac{1}{4} \delta[n-3] \]

- OUTPUT is called “IMPULSE RESPONSE”

\[ h[n] = \{ \ldots, 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, \ldots \} \]
4-pt Avg Impulse Response

\[ y[n] = \frac{1}{4} (x[n] + x[n - 1] + x[n - 2] + x[n - 3]) \]

\( \delta[n] \) “READS OUT” the FILTER COEFFICIENTS

\[ h[n] = \{ ..., 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, ... \} \]

“h” in \( h[n] \) denotes Impulse Response

NON-ZERO
When window overlaps \( \delta[n] \)
FIR Impulse Response

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

\[
x[n] = \delta[n]
\]

\[
y[n] = h[n]
\]

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n &lt; 0 )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>( \ldots )</th>
<th>( M )</th>
<th>( M + 1 )</th>
<th>( n &gt; M + 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x[n] = \delta[n] )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( y[n] = h[n] )</td>
<td>0</td>
<td>( b_0 )</td>
<td>( b_1 )</td>
<td>( b_2 )</td>
<td>( b_3 )</td>
<td>( \ldots )</td>
<td>( b_M )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
y[n] = \sum_{k=0}^{M} b_k x[n-k]
\]

\[
y[n] = \sum_{k=0}^{M} h[k] x[n-k]
\]

CONVOLUTION
Filtering Example

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

- 3-point AVERAGER
  - Changes A slightly

\[ y_7[n] = \sum_{k=0}^{6} \left( \frac{1}{7} \right) x[n - k] \]

\[ y_3[n] = \sum_{k=0}^{2} \left( \frac{1}{3} \right) x[n - k] \]
3-pt AVG EXAMPLE

Input: \( x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \) for \( 0 \leq n \leq 40 \)
7-pt FIR EXAMPLE (AVG)

Input: \(x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)\) for \(0 \leq n \leq 40\)
Next week

Section 5-4
Section 5-5
Section 5-6
Section 5-7

That's all Folks!